Lunar Material Transport: Comparison of Modes Employing Achromatic Trajectories

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Introduction

A RECENT series of papers[1-3] has given a theoretical treatment of achromatic trajectories and has explored their use in the industrial-scale transport of lunar resources. The transport mode treated involves launch from a lunar nearside site to the L2 (translunar) libration point, with subsequent transfer to a stable Earth orbit in 2:1 resonance with the Moon. However, it has not been considered whether there may exist other transport modes. Such modes would consist of launch from the lunar surface to any of the libration points L1, L2, L4 and L5, these being equilibrium points suitable for location of a mass-catcher. (L3 is not of interest, since it is hard to reach from the Moon and far from any suitable orbit for a space manufacturing facility.) Thus, this paper presents a survey and comparison of such modes.

One may employ achromatic trajectories when launching lunar material on ballistic (uncorrected) trajectories to a point P in space from a launch site on or near the lunar equator at longitude λ_0 with velocity V_T tangent to the lunar surface. An error ΔV_T leads to a miss distance at P of Δy:

$$\Delta y = \Delta y_0 + C_{VT}\Delta V_T + C_{VT2}(\Delta V_T)^2 + C_{VT3}(\Delta V_T)^3 + \cdots \quad (1)$$

With both λ_0 and V_T as free parameters, one may drive both Δy_0 and C_{VT} to zero. The trajectory then is achromatic. Achromatic trajectories permit large increases in the allowable ΔV_T for fixed Δy. Thus, when launching to L2 using achromatic trajectories, $\Delta V_T = 10$ cm/s gives $\Delta y = 20$ m.[1] Using nonachromatic trajectories,[4] one requires $\Delta V_T = 4$-20 microns/s for the same Δy.

Determination of Solutions

The problem considered is to identify all values of (λ_0, V_T) for which achromatic trajectories exist having $C_{VT} = 0$ at any of L1, L2, L4, and L5. Reference 1 gives examples of achromatic trajectories both in the circular and in the elliptic planar-restricted three-body problems; additionally, theorems are given[1] which prove that achromatic trajectories continue to exist when one introduces perturbations due to the Sun and to other bodies. A treatment of three-dimensional effects[1] shows them, ordinarily, to pose no practical impediment to the use of achromatic trajectories. Hence, it is legitimate to use the circular-restricted three-body problem; in Moon-centered coordinates, the equations treated are

$$\ddot{x} - 2\dot{y} = \Omega_x \qquad \ddot{y} + 2\dot{x} = \Omega_y$$
$$\Omega = \tfrac{1}{2}[(x+1-\mu)^2 + y^2] + (1-\mu)/r_1 + \mu/r_2$$
$$r_1^2 = (x+1)^2 + y^2 \qquad r_2^2 = x^2 + y^2 \quad (2)$$

The units are normalized: unit mass = (Earth + Moon) so that lunar mass $\mu = 0.01215$, unit distance = 384,410 km, unit time = 104.362 h, and unit velocity = 1023.17 m/s. The lunar radius is $r_m = 0.00452133$. The x axis is along the Earth-Moon line of centers; the y axis points in the direction of the Moon's motion.

Points where C_{VT} equals 0 are known as focus points. Two methods were used for finding such points:

1) *Method of differences.*[1] One integrates Eqs. (2) simultaneously for two independent sets of initial conditions:

$$\begin{aligned} x_1(0) &= -r_m\cos\lambda_0 & x_2(0) &= x_1(0) \\ \dot{x}_1(0) &= (V_T - \epsilon/2)\sin\lambda_0 & \dot{x}_2(0) &= (V_T + \epsilon/2)\sin\lambda_0 \\ y_1(0) &= -r_m\sin\lambda_0 & y_2(0) &= y_1(0) \\ \dot{y}_1(0) &= -(V_T - \epsilon/2)\cos\lambda_0 & \dot{y}_2(0) &= -(V_T + \epsilon/2)\cos\lambda_0 \end{aligned} \quad (3)$$

where ϵ is a small error. At each integration step, one computes Δt_1, Δt_2, satisfying

$$x_1 + \dot{x}_1\Delta t_1 = x_2 + \dot{x}_2\Delta t_2 = x_c$$
$$y_1 + \dot{y}_1\Delta t_1 = y_2 + \dot{y}_2\Delta t_2 = y_c \quad (4)$$

A focus point exists at (x_c, y_c) when $\Delta t_1 + \Delta t_2 = 0$. Then, as shown in Ref. 1,

$$C_{VT2} = \frac{\dot{x}_2\dot{y}_1 - \dot{x}_1\dot{y}_2}{(\dot{x}_1^2 + \dot{y}_1^2)^{1/2}} \cdot \frac{\Delta t_2 - \Delta t_1}{\epsilon^2} \cdot 0.03672 \qquad \frac{\text{m}}{(\text{cm/s})^2} \quad (5)$$

With C_{VT2} in m/(cm/s)^2, $C_{VT2} = \Delta y$, in meters, due to $\epsilon = 1$ cm/s.

2) *Method of integration.* One integrates Eqs. (2) subject to initial conditions:

$$x(0) = -r_m\cos\lambda_0 \qquad \dot{x}(0) = V_T\sin\lambda_0$$
$$y(0) = -r_m\cos\lambda_0 \qquad \dot{y}(0) = -V_T\cos\lambda_0 \quad (6)$$

Simultaneously, one integrates the linear variational equations:

$$\ddot{\xi} - 2\dot{\eta} = \Omega_{xx}\xi + \Omega_{xy}\eta \qquad \ddot{\eta} + 2\dot{\xi} = \Omega_{yx}\xi + \Omega_{yy}\eta \quad (7)$$

subject to initial conditions

$$\xi(0) = 0 \quad \dot{\xi}(0) = \sin\lambda_0 \quad \eta(0) = 0 \quad \dot{\eta}(0) = -\cos\lambda_0 \quad (8)$$

Then (ξ,η) is a vector of displacement due to errors in V_T. The focus point exists where (ξ,η) is parallel to $(\dot{x}, \dot{y})$; that is, where

$$\xi\dot{y} - \eta\dot{x} = 0 \quad (9)$$

Equations (7) and (9) also define the condition for optimality of an impulsive transfer.[5] However, in treating optimal trajectories, Eqs. (7) are integrated not as an initial-value problem but as a two-point boundary-value problem:

$$\xi(0) = \sin\lambda_0 \quad \eta(0) = -\cos\lambda_0 \quad \xi(t_f) = -\cos\theta \quad \eta(t_f) = \sin\theta \quad (10)$$

where t_f is the flight time to point P (the target), and θ is the orientation of the velocity vector measured clockwise from the negative x axis.

Those values of (λ_0, V_T) yielding focus points near any one of L1, L2, L4, L5 were then used to initialize a Newton iteration which sought to obtain (λ_0, V_T), yielding a focus point coincident with a libration point. This was found to be a most delicate operation in that the location of (x_c, y_c) was often quite sensitive to small changes in λ_0 or V_T. Hence, before achieving confidence that the roster of these critical

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Table 1 Determination of critical launch points

Mode	Method of integration		Method of differences, $\epsilon = 10^{-5}$		Method of differences, $\epsilon = 10^{-3}$		Lunar Orbiter photographic reference
	λ_0	V_T	λ_0	V_T	λ_0	V_T	
D1L1	209°.518978	2.27692798	209°.611698	2.27705672	209°.613728	2.27707130	I-28M
R2L1	282°.796555	2.29372144	282°.796448	2.29372472	282°.786453	2.29372567	IV-173 H_3
D1L2	33°.100310	2.28509620	33°.110759	2.28507483	33°.112081	2.28512156	I-41M
R2L2	112°.976802	2.29618573	112°.977218	2.29619041	112°.971572	2.29619208	II-196M
D1L4	183°.269058	2.31179401	183°.269207	2.31179410	183°.161100	2.31178293	II-33M
D2L4	312°.648701	2.31309688	312°.648891	2.31307903	311°.503525	2.31237909	IV-150 H_1
R1L4	278°.575283	2.31289178	278°.575302	2.31289217	277°.921722	2.31251922	IV-181 H_3
D1L5	336°.324865	3.03346044	336°.326848	3.03403899	336°.324944	3.03398088	IV-126 H_1
D3L5	350°.015865	2.31123853	350°.015865	2.31123802	349°.838417	2.31117970	IV-114 H_1
D3L5-A	351°.865395	2.32973859	351°.860600	2.33006406	351°.862857	2.32972574	IV-114 H_1

Table 2 Reference achromatic trajectories: parameters[a]

Mode	θ deg	V	t_f	C_{VT2}, m/(cm/s)^2	C_{VN}, m/(cm/s)	C_{VZ} m/(cm/s)	C_λ	C_Z
D1L1	−65.7500	0.2159	0.4161	0.0398	445.102	14.618	30.3581	−28.9577
R2L1	−14.8096	0.3513	1.9260	55.0810	941.490	−1.994	69.2973	−29.7057
D1L2	115.3931	0.2602	0.4260	0.1901	511.488	32.163	35.8640	−33.4716
R2L2	167.0382	0.3443	2.1460	61.6067	880.473	−0.065	54.4949	−31.7670
D1L4	101.2952	0.0789	3.3906	329.3701	13,926.199	−107.011	821.7234	74.8564
D2L4	124.6659	0.1108	4.4906	1405.5807	18,935.832	54.840	1098.0670	−1.2751
R1L4	127.3086	0.1066	4.2807	867.3198	11,344.900	56.341	646.1748	4.2976
D1L5	−119.4586	1.9588	0.5310	0.0153	2007.687	1136.186	103.1819	−90.3993
D3L5	38.5679	0.0513	4.9206	986.7370	17,425.359	−79.318	1114.1430	58.8968
D3L5-A	−38.5036	0.3013	3.0554	24.2035	11,518.129	100.672	768.0997	−70.4260

[a] Units are dimensionless unless otherwise specified.

values of (λ_0, V_T) was complete, it was necessary to test some 34,000 trial values of (λ_0, V_T). Integrations were carried to $t \sim 7$, with λ_0 being taken from 0-360 deg; the cases tested were as follows:

1) V_T every 0.005 from 2.26-2.36; λ_0 every 4 deg; direct (eastward) launch; method of differences with $\epsilon = 0.001$.
2) V_T every 0.0025 and $\epsilon = 0.00001$; otherwise same as 1.
3) Method of integration; otherwise the same as 2.
4) V_T from 2.36-2.56; otherwise the same as 2.
5) V_T every 0.10 from 2.5-3.5; λ_0 every 18 deg; otherwise the same as 2.
6) V_T every 0.001 from 2.27-2.32; λ_0 every 4 deg; retrograde (westward) launch; method of differences, $\epsilon = 0.001$.
7) Same as 6 except $\epsilon = 0.00001$.
8) Same as 6 with method of integration.
9) Same as 7 except V_T from 2.32-2.52.
10) Same as 5 except with retrograde launch.

Ten such critical launch sites were found; their parameters (λ_0, V_T) are listed in Table 1. Also listed in Table 1 for each site are one or more references to Lunar Orbiter site photographs. These constitute modes for lunar mass transport; each mode consists of a launch site together with its associated achromatic trajectory to its target libration point. To designate these modes, there is a four-character code: $A_l mLn$. A_l is the letter D or R representing direct or retrograde launch, m is the sequential focus-point number, i.e., the focus point of interest is the mth which appears in time sequence; and Ln ($n = 1$, 2, 4, or 5) is the target libration point. The transport mode treated in Refs. 1-4 is then designated D1L2; this is read "direct launch, first focus point in sequence, launch to L2."

Comparison of Modes

For the launch modes of Table 1, Table 2 gives parameters θ, V $[=(\dot{x}_f^2+\dot{y}_f^2)^{1/2}]$, C_{VT2}, t_f, all at the target point. Also given are the additional sensitivity coefficients C_{VN}, C_λ, C_{VZ}, C_Z. C_{VN} is the miss distance at the target, in meters, due to $\Delta V_N = 1$cm/s, where ΔV_N is the component of launch velocity error normal to the lunar surface. C_λ is the "magnification" of the error $r_m \Delta\lambda_0$ at launch: a displacement in launch location by $r_m \Delta\lambda_0$ is multiplied C_λ-fold at the target. C_{VZ} and C_Z are the analogous coefficients associated respectively with an out-of-plane component of launch velocity error, ΔV_Z, and with an out-of-plane displacement Δz. In particular, as discussed in Ref. 1, C_{VZ} permits assessment of whether three-dimensional effects may pose difficulties in the real-world use of achromatic trajectories.

A desirable launch mode would offer a launch site on the lunar nearside and in mare terrain, preferably in the zone of $300 \le \lambda_0 \le 60$ deg which has been extensively mapped and studied by Apollo missions. Low V is desirable, so as to ease the problem of catching the lunar-launched payloads and maneuvering the mass-catcher.[3] Low values for C_{VT2}, C_{VN}, and C_{VZ} are of great importance in permitting large tolerances on the allowed ΔV_T, ΔV_N, ΔV_Z at launch.

From these considerations, D1L2 is found to be preferable, all other modes having one or more serious difficulties by comparison. The difficulties are as follows:

D1L1 – farside location (near Korolev Basin)
R2L1 – high C_{VT2}
R2L2 – farside location, extremely rugged terrain, high C_{VT2}
D1L4 – farside location, extremely rugged terrain, very high C_{VT2}, C_{VN}
D2L4 – very high C_{VT2} and C_{VN}
R1L4 – very high C_{VT2} and C_{VN}
D1L5 – very high V; high C_{VN} and C_{VZ}
D3L5 – very high C_{VT2} and C_{VN}
D3L5-A – high C_{VT2}, very high C_{VN}

However, the two modes to L1 need not be rejected out of hand. D1L1, except for its location, offers parameters even more favorable than D1L2. Korolev Basin is generally pocked with kilometer-sized craters, unlike the smooth lava-filled basins constituting nearside maria; but detailed photographic studies may disclose isolated sites smooth enough to permit construction. R2L1, though requiring a closer restriction on allowed ΔV_T, may be able to take advantage of the nearby crater Riccioli for site location, since that crater has a smooth marelike floor.

Deprit and Henrard[6] have shown that in the Earth-Moon system, bodies leaving L1 with near-zero velocity follow a 5:2 resonant orbit. This is also very nearly true[3] for bodies departing L2 with small velocity. Hence the arguments of Ref. 3, proposing that if mass-catching is done near L2 then the manufacturing facility should be in a 2:1 resonant orbit, apply as well when catching is done near L1.

The popular legend of an L5 space colony finds little support from the present results. While one may arbitrarily adopt decision criteria favorable to the D3L5 or D3L5-A mode, such criteria can appear only as ad hoc assumptions. When using a mass-driver and mass-catcher in lunar material transport, the difficulty is the sensitivities at launch; from this perspective, the launch modes to L4 or L5 are entirely unsuitable. Nor is it desirable to transport mass from L2 to L4 or L5.[3] Hence, any proposal to locate a space colony at L5 must involve an alternate mode for resource transport, e.g., the use of rockets. Such a proposal must then show advantage for L5 vs such alternatives as lunar orbit, geosynchronous orbit, or a stable "giant-halo" orbit of L1 or L2,[7] all of which may be conveniently reached by rockets.

Conclusions

An evaluation of some 34,000 test achromatic trajectories has yielded ten well-defined modes for lunar material transport. These have been compared from the standpoints of arrival velocity, launch side topography and location, and sensitivities with respect to errors in components of launch velocity. This comparison favors the D1L2 mode treated in previous papers.[1-4] However, the possibility of using relatively smooth terrain in the Korolev Basin and in the crater Riccioli suggests that the D1L1 or R2L1 modes may prove usable. All other modes involve high arrival velocity, very rugged launch-site terrain (often on the lunar farside), or high-launch sensitivities, and thus presently appear inferior to D1L2.

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Flight Instability Produced by a Rapidly Spinning, Highly Viscous Liquid

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A NEW type of flight instability has been observed for spin-stabilized projectiles filled with a liquid whose viscosity is approximately 100,000 times larger than water. The stability of liquid-filled projectiles is a well documented problem. Stewartson[1] provided a description of one physical mechanism for instability as a matching of a natural frequency of oscillation of the liquid (an eigenfrequency) and the fast frequency of motion of the projectile and calculated the eigenfrequency spectrum and the amplitude behavior of the carrier vehicle for an inviscid liquid. Wedemeyer[2] then provided viscous boundary-layer corrections to the Stewartson solution. The range of application of these corrections is controlled by the Ekman number ($E=\nu/a^2p$, where ν is the kinematic viscosity of the liquid, a is the radius of the cylindrical payload compartment, and p is the spin of the projectile). Good agreement between the Wedemeyer theory and a liquid-filled gyroscope is obtained for $10^{-4}>E>10^{-6}$.[3] The Stewartson-type instability directly depends upon the liquid fill ratio and the geometry of the payload compartment as any eigenvalue problem would.

Such is not the case for a recently observed instability which is apparently controlled by the viscosity of the liquid when E is much larger. Laboratory experiments with a spin fixture stimulated interest in high Ekman number flows. The laboratory spin fixture causes a full-scale cylindrical payload container for a 155-mm projectile to undergo simultaneous spin and precession and measures the canister despin due to the action of a nonrigid payload fill. Spin fixture results and flight data have shown good qualitative agreement on a variety of nonrigid payload configurations.[4] A series of spin fixture tests were conducted with cylindrical containers completely filled with homogeneous liquids of various viscosities. Figure 1 shows spin fixture despin moment data as a function of liquid kinematic viscosity. Note that the despin moment approaches zero at both low and high viscosity extremes and achieves the largest value at a kinematic viscosity of about 100,000 centistokes. Subsequently, a series

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